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ON THE INNER PROBLEM IN THE CASE OF A CURVED SPACE-TIME OF SPHERICAL SYMMETRY

by

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Relativity -- on the inner problem in the case of a curved space-time of spherical symmetry.

Note submitted by G. Darmois

The external problem solved independently by various authors[1] /1276
admits, as we know, the following solution where α, λ are two constants:

$$ds^2 = U_4^2\, dx_4^2 - dx_1^2 - U_2^2 \left[dx_2^2 + \sin^2 x_2\, dx_3^2\right],$$

$$dx_1 = \frac{dr}{\sqrt{1 - \frac{\alpha}{r} - \frac{\lambda}{3} r^2}}, \quad U_4^2 = 1 - \frac{\alpha}{r} - \frac{\lambda}{3} r^2, \quad U_2^2 = r. \tag{1}$$

The internal problem of the Schwarzschild ds^2 has been solved for a special case by Schwarzschild himself, and later generalized by L. Brillouin. The general solution of Brillouin and T. de Donder has been resurrected in a simpler form by Haag.

The internal problem, in the new case where $\lambda \neq 0$, was solved by Maurice Nuyens[2] using the methods recommended by T. de Donder. Haag showed that his elegant solution was still applicable[3].

Schwarzschild's solution has provoked some interesting remarks and criticisms on the part of Eddington[4].

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[1] Weyl, Eddington, Chazy.

[2] Comptes Rendus, May 14, 1923.

[3] Comptes Rendus, May 28, 1923.

[4] The mathematical theory of relativity, p. 169-170.

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These observations, which seem to be justified, deprive Schwarzschild's results of some of their physical significance. In particular, for the case of large material spheres, Eddington estimates that the conclusions advanced lack foundation. These criticisms /1277 apply to the solutions to which we just referred.

It is remarkable that the internal problem admits a very simple, one might say a very familiar, solution. To find this solution, we take the material tensor in the form $T^{\alpha\beta} = \rho_0 u^\alpha u^\beta$, the invariant $T = \rho_0$ being a constant for an incompressible fluid. We suppose ρ_0 to be constant for the solution to follow.

For greater clarity, we take the internal problem as our point of departure. First we adopt, as our space coordinates, the three constants of a line of flow, as our time coordinate the interval x_4 counted along one of those lines. The fundamental form is then

$$dx_4^2 + 2g_{i4}\, dx_i\, dx_4 + g_{\lambda\mu}\, dx_\lambda\, dx_\mu \quad (i, \lambda, \mu = 1, 2, 3).$$

The g_{i4} are, in the general case, independent of x_4. The $g_{\lambda\mu}$ are in general dependent upon x_4. But we are seeking a particular solution for which $g_{i4} = 0$, with $g_{\lambda\mu}$ independent of x_4. Under these conditions, one very simple solution is Einstein's solution

$$dx_4^2 - dx_1^2 - R^2 \sin^2 \frac{x_1}{R} [dx_2^2 + \sin^2 x_2\, dx_3^2] \quad \left(\frac{1}{R^2} = \frac{\varkappa \rho_0}{2} \right), \qquad (2)$$

where $\varkappa$ is the ordinary constant in Einstein's gravitational equations.

Ordinarily, we view this solution as one extending throughout the domain of variation of $x_1 x_2 x_3$ which could be obtained by letting

x_3 vary from 0 to 2π, x_2 from 0 to π, $\frac{x_1}{R}$ from 0 to π. That is what is known as Einstein's universe.

The curves on which x_1 alone varies are the geodesics in the space, normal to the geodesic spheres x_1 = const, of which the two centers are $x_1 = 0$, $x_1 = R\pi$.

Let us restrict our treatment to the portion of space around the point $x_1 = 0$ and bounded by $x_1 = \underline{a}$, and let us assume matter to be limited to that portion of space. The extension of the inner ds^2 by an outer ds^2 is then well defined. It is natural to think that this extension will provide a ds^2 of form (1).

Indeed, if we impose the continuity conditions of the potentials and their first derivatives with respect to the variable x_1, the geodesic distance normal to the separation space, we find the necessary and sufficient conditions

$$\lambda = \frac{1}{R^2}, \quad \alpha = \frac{2}{3} R \sin^3 \frac{\alpha}{R}, \quad r_0 = R \sin \frac{\alpha}{R}.$$

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